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Abstract—**As bitcoin becomes more important as a worldwide financial phenomenon, it also becomes important to understand its sources of value formation. There are three ways to obtain bitcoins: buy them outright, accept them in exchange, or else produce them by 'mining'. Mining employs computational effort which requires electrical consumption for operation. The cost of electricity per kWh, the efficiency of mining as measured by watts per unit of mining effort, the market price of bitcoin, and the difficulty of mining all matter in making the decision to produce. Bitcoin production seems to resemble a competitive market, so in theory miners will produce until their marginal costs equal their marginal product. Break-even points are modeled for market price, energy cost, efficiency and difficulty to produce. The cost of production price may represent a theoretical value around which market prices tend to gravitate. As the average efficiency increases over time due to competition driving technological progress – as inefficient capital becomes obsolete it is removed while new capital replaces them – the break-even production cost of bitcoins denominated in dollars will fall. Increased efficiency, although necessary to maintain competitive advantage over other miners could serve to drive the value of bitcoin down, however adjustments in the mining difficulty and the regular halving of the block reward throughout time will tend to counteract a decreasing tendency in cost of production.**

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JEL Classification: C51, D58, E42, E47, G12

I. Introduction

There are three primary ways one can obtain bitcoins (BTC), the most popular and widely accepted of the so-called cryptocurrencies. The first is to buy them directly from another individual or via an online marketplace in exchange for national fiat currencies or other cryptocurrencies.[1] The second way is to accept them as payment for goods and services, or as wages. The third is to 'mine' for bitcoins using computer hardware and software specifically designed to solve the cryptographic algorithm underlying the bitcoin protocol, thus producing new bitcoins.[2]

The decision to mine for bitcoin comes down to profitability. A rational agent would not undertake production of bitcoins if they incurred a real loss in doing so. Bitcoin mining employs computational effort, measured in gigahashes per second (GH/s). The hashrate, or number of hashes per second can be thought of as somewhat analogous to the cycles per second (hertz) of computer processors. This computational effort is directed at mining bitcoin, in competition with many other miners who presumably are also motivated by profit, on average. The more powerful the mining effort (the higher the hashrate), the more likely it is to successfully mine bitcoins during a given interval.[3]

The success in finding bitcoins depends not only on the hashing power, but also on the difficulty level of the algorithm at the time that mining is undertaken. The difficulty specifies how hard it is to find a bitcoin during some interval, the higher the difficulty the more computational effort will be required to mine bitcoins at the same rate as with a lower difficulty setting. The bitcoin network automatically adjusts the difficulty variable so that one block of bitcoins is found, on average, every ten minutes. As more aggregate computational effort is added to mining bitcoins, the time between blocks will tend to decrease below ten minutes, the result being that the network will adjust the difficulty upwards to maintain the set ten minute interval accommodating the excess mining effort. Likewise, if mining effort is removed from the network, the length between blocks would grow longer than ten minutes and the network will adjust the difficulty downwards to restore the ten minute interval.

Each unit of mining effort has a fixed sunk cost involved in the purchase, transportation and installation of the mining hardware. It also has a variable, or ongoing cost which is the direct expense of electricity consumption. Each unit of hashing power consumes a specific amount of electricity based on its efficiency, which has a real-world cost for the miner. Because miners cannot generally pay for their electricity cost in bitcoin, they must refer to the currency price of a bitcoin to measure profitability given a monetary cost of electricity.

1 Alternative cryptocurrencies are collectively referred to as 'altcoins'.

2 This mining algorithm is known as SHA-256. For more on the technical specifications of bitcoin mining, please refer to: http://en.wikipedia.org/wiki/Bitcoin_network#Bitcoin_mining

3 The typical interval used for mining profitability calculations is currently bitcoins earned per day.

It seems to be the case that the marginal cost of bitcoin production matters in value formation. Instead of approaching bitcoin as a digital money or currency, it is perhaps more appropriate to consider it a virtual commodity with a competitive market of producers.

Taken together, the important variables in forming the decision to mine are: [1] the cost of electricity, measured in cents per kilowatt-hour; [2] the energy consumption per unit of mining effort, measured in watts per GH/s, a function of the cost of electricity and energy efficiency;[4] [3] the monetary price of bitcoin in the market; and [4] the difficulty of the bitcoin algorithm.[5] An individual would undertake mining if the marginal cost per day (electricity consumption) were less than or equal to the marginal product (the number of bitcoins found per day on average multiplied by the dollar price of bitcoin). If bitcoin production is a competitive commodity market, albeit a virtual one, then we would theoretically expect marginal cost to equal marginal product.[6]

The main cost in bitcoin mining is the energy consumption which is needed to facilitate the computational labor employed in mining.[7] The market price is determined by the supply and demand for bitcoin at any given moment, while the cost of production might set a lower bound in value around which miners will decide to produce or not.

Of course, there are likely to be many subjective motivations for bitcoin mining beyond the objective components elaborated in this paper. Individual decision makers may operate regardless of cost if they believe that there is enough speculative potential to the upside. Bitcoin mining may draw in those who find the features of anonymity and lack of governmental oversight attractive. Some miners may decide to hoard some or all of their lot and not regularly engage in offering mined bitcoins in the open market, a sort of bitcoin 'fetishism'. Some miners may be subject to an opportunity cost whereby it would be more profitable to expend the same electrical capacity for some other pursuit. Subjective rationales for mining may induce some individuals to make the decision to produce at a marginal loss for prolonged periods of time. The speculative and money-like properties of bitcoin, as a means of exchange and a potential store of value, add a subjective portion to any objective attempt at forming an intrinsic value. New and innovative uses of the bitcoin network for non-bitcoin specific applications are also likely to add value for mining.

4 A Watt per GH/s is equivalent to a Joule per GH (1 W/GH/s = 1 J/GH)

5 The block reward also matters, but this value changes only after longer intervals. More about this feature is provided in the discussion section.

6 Which would also equal selling price, in theory.

7 Other much smaller costs include internet service, hardware maintenance, computer cables etc.

II. Brief Survey of Related Literature

Yermack (2013) has put forth that bitcoin has no intrinsic value. If this were the case, there would be no incentive to mine and produce bitcoins except as a speculative venture. While speculation may be the case for certain individual miners, the mining ecosystem is now so large that producing bitcoins must be profitable on an on-going basis and not solely with the hope of some sufficiently sizable payoff in the far future. If an individual thought the market price would skyrocket, they could simply buy bitcoins in the open market and not deal with obtaining, installing and maintaining a mining operation.

Hanley (2013) argues that the value of bitcoin merely floats against other currencies as a pure market valuation with no fundamental value to support it. Woo, et al. (2013) proposes that bitcoin may have some fair value due to its money-like properties as a medium of exchange and a store of value, but without any other underlying basis.

Hayes (2014) undertook a cross sectional data analysis of market prices for a number of cryptocurrencies relative to that of bitcoin. He found that variations in relative values among cryptocurrencies take place at the margin, and that differences in the rates of unit production matter.

Garcia, et al. (2014) asserts that the cost of production through mining does matter in coming up with a fundamental value for bitcoins insofar as it represents a lower bound. This paper will elaborate on that general idea and formalize it to identify a cost of production model for bitcoin. Doing so can identify theoretical break-even levels in market price, electricity cost, mining energy efficiency, and mining difficulty for individual miners – and may be extended to impute averages for the aggregate network.

While it may be tempting to objectify these results to impute a true intrinsic value for bitcoin, I would caution against making such a leap. Even if the models developed in this paper can theoretically determine an intrinsic value, extreme volatility and frequent market price fluctuations in the few years since bitcoin has been around could make identifying such an intrinsic value meaningless in application. There is also the matter of subjective components of value formation which are more difficult to quantify.

III. The Decision To Mine & The Cost of Production

The objective decision to mine for bitcoins can be modeled. The necessary inputs are the dollar price of electricity, the energy consumption per unit of mining power, the dollar price of bitcoins, and the expected production of bitcoins per day which is based in part on the mining difficulty.[8]

8 For illustrative purposes only, the US dollar will be the currency used to price bitcoin. In reality, there are bitcoin miners worldwide, notably in Russia, Europe, and China who will buy electricity in their regional currency and at their local rate.

Hayes (2015) applied a model for determining the expected number of cryptocurrency coins to be mined per day on average given the difficulty and block reward (number of coins issued per successful mining attempt) per unit of hashing power.

$$\text{BTC/day}^* = [(\beta \cdot \rho)/(\delta \cdot 2^{32})/\text{sec}_{\text{hr}}] \cdot \text{hr}_{\text{day}} \qquad (1)$$

where BTC^*/day is the expected amount of bitcoins a miner can expect to earn per day, β is the block reward, ρ is the hashing power employed by a miner, and δ is the difficulty.[9,10] The constant sec_{hr} is the number of seconds in an hour, 3,600. The constant hr_{day} is the number of hours in a day, 24. The constant 2^{32} is the normalized probability of a single hash solving a block and is a function of the bitcoin algorithm.[11] The value for ρ, for the purposes of this paper, will be given at a standard unit of 1,000 GigaHashes per second (GH/s) of mining power (or equivalently 1 TeraHash per sec, (TH/s)). The block reward for bitcoin is given at 25 bitcoins per block, currently. The constants which normalize the dimensional space for daily time and for the mining algorithm can be summarized by the term θ, which will equal: $\theta = 24\text{hr}_{\text{day}} \cdot 2^{32}/3600\text{sec}_{\text{hr}} = 28{,}633{,}115.30667$. Equation (1) can thus be rewritten:

$$\text{BTC/day}^* = \theta\,(\beta \cdot \rho)/(\delta) \qquad (2)$$

For example, the number of bitcoins one can expect per day employing 1,000 GH/s and with a difficulty of 47,427,554,950.6483 would be calculated with equation (2) to be 0.010604 BTC^*/day.[12]

The cost of mining per day, E_{day} can be expressed as:

$$E_{\text{day}} = (\text{price per kWh} \cdot 24\ \text{hr}_{\text{day}} \cdot \text{W per GH/s})(\text{GH} / 1{,}000) \qquad (3)$$

The marginal product of mining should theoretically equal its marginal cost in a competitive market, which should also equal its selling price. Because of this theoretical equivalence, and since cost per day is expressed in $/day and production in BTC/day, the $/BTC price level is simply the ratio of (cost/day) / (BTC/day). This objective price of production level, p^*, serves as a logical lower bound for the market price, below which a miner would operate at a marginal loss and presumably remove them self from the network. p^* is expressed in dollars per bitcoin, given the difficulty and cost of production:

$$p^* = E_{\text{day}} / (\text{BTC/day}^*) \qquad (4)$$

Note that since equation (4) contains equation (2) in the denominator, p^* is a function of the difficulty and block reward.

Given an observed market price (p) and a known difficulty, one can solve for the break-even electricity cost in kilowatt-hours:

$$\text{price per kWh}^* = [p(\text{BTC/day}^*)/24\text{hr}_{\text{day}}] / \text{W per GH/s} \qquad (5)$$

Given a known cost of production and observed market price, one can solve for a break-even level of mining difficulty:

$$\delta^* = (\beta \cdot \rho \cdot \text{sec}_{\text{hr}} \cdot \text{hr}_{\text{day}}) / [(E_{\text{day}}/p) \cdot 2^{32}] \qquad (6)$$

And, to solve for a break-even energy efficiency, we can again rearrange terms given a market price, cost of electricity per kilowatt-hour, and difficulty:

$$\text{W per GH/s}^* = (p \cdot \text{BTC/day}^*)/(\text{price per kWh} \cdot 24\text{hr}_{\text{day}}) \qquad (7)$$

IV. Discussion

These equations are useful in application as well an in theory. It informs miners objectively as to which price they should undertake or else give up mining. It also informs miners when to stop or start mining given changes in difficulty and electricity costs. Furthermore, looking at market prices for a given difficulty and known average electricity costs, the average energy efficiency of mining for the entire network can be imputed.

It is useful to consider a hypothetical example: Assume that the average electricity cost for the world is 11.5 cents per kilowatt-hour and the average energy efficiency of ASIC mining hardware currently deployed is 0.95 J/GH. The average cost per day for a 1,000 GH/s (1 TH/s) mining rig would be (0.115 · 24 · 0.95) · (1,000 / 1,000) = $2.622/day. The number of bitcoins that 1,000 GH/s of mining power can find in a day with a difficulty of 47,427,554,951 is 0.010604 BTC/day. Because these two values are theoretically equivalent, to express them in dimensional space of $/BTC we simply take the ratio (2.622 $/day) / (0.010604 BTC/day) = $247.27/BTC. This is surprisingly close to the current market value of around $255-$260/BTC.[13]

If the market price were to drop below that value, miners would be operating at a marginal loss and halt production. Continuing the analysis of this example, if the difficulty were to increase to greater than 57,541,669,370, holding all else constant, miners would cease operations. Also in this example, and holding all else constant, miners would cease operations if their energy costs rose to more than 13.952 cents per kilowatt-hour. Likewise, a miner would cease operations if their mining hardware consumed energy at an efficiency worse than 1.15 W per GH/s. These figures are hypothetical for the purposes of elaborating the applicative

9 Block Reward is expressed in the units: BTC/block

10 Difficulty is expressed in the units: GH/block

11 Bitcoin relies on the SHA-256D encryption algorithm which miners try to solve. Successfully solving this algorithm first results in the system awarding that miner with a block of bitcoins.

12 The difficulty value represents the mining difficulty at the time of writing this paper. The actual mining difficulty varies through time as aggregate mining effort is added or removed from the network.

13 Market price observable on March 19, 2015

usage of the equations introduced above, but have been chosen to be fairly close to current real-world practical averages.

As real-world mining efficiency increases, which is a likely result of competition, the break-even price for bitcoin producers will tend to decrease. Low cost producers will compete in the marketplace by offering their product at lower and lower prices. Mining hardware energy efficiency has already increased greatly since the days of CPU or GPU mining. A research study found that the average mining efficiency over the period 2010-2013 was a staggering 500 Watts per GH/s (Garcia, et al., 2013). Today, the best ASIC mining rigs available for purchase have somewhere around 0.50 – 0.60 Watts per GH/s energy efficiency. The average energy efficiency right now across the mining network, which is the value which regulates the marginal cost, seems to be around 0.90 – 1.00 Watts per GH/s.[14] This speaks to the rapid pace of technological advancement produced over the past few years and months in mining energy efficiency. The bitcoin mining network is vast in size and scope and it is likely that some miners are at work with hardware that is older and less efficient than the best available.

Bitcoin mining, unlike traditional commodity production, has the unique feature of a regular difficulty adjustment in order to maintain a steady rate of unit production over time – specifically, a block of bitcoins will be mined on average once every ten minutes, regardless of aggregate mining power. Unlike most produced commodities where the supply can change to accommodate fluctuations in demand, the supply of bitcoin is hardwired at its steady rate with the difficulty setting adjusting up and down to maintain that linear rate of production through time.

The difficulty adjustment acts as a stabilizing mechanism, increasing the cost of production; as more aggregate mining power is brought on line, the mining difficulty increases. For example, if a mining rig can find 1 BTC/day on average with today's difficulty, the same rig can expect to produce less per day when the difficulty increases 10% or 20% etc. If miners are not able to supply enough new coins to meet an influx of new demand, the market price can see increases while the cost of production remains largely the same. This would induce miners to increase their mining efforts which would then cause the difficulty to increase, raising the cost of production until presumably a new break-even level is reached. This mechanism tends to counteract the downward tendency caused by increasing energy efficiency.

One final insight that could have sizable consequences for the cost of production of bitcoin relates to the block reward amount and how changes in this variable will impact BTC/day production. When bitcoin was launched, each block mined was composed of 50 bitcoins. That amount is set to halve every four years, and in 2012 the block reward became 25.[15] The block reward will again halve to 12.5 bitcoins per block, expected mid-September, 2016, and will again in the year 2020, and so on. If we refer back to the illustrative example above and substitute a 12.5 BTC block reward for the current 25, the expected BTC^{*}/day' becomes half of 0.010604, or 0.0005302 per 1,000 GH/s. Using the hypothetical example above and given this new BTC^{*}/day', the break-even price for a bitcoin would increase suddenly to \$494.54, holding all else constant.[16] If the market price of bitcoin does not increase in turn, it will suggest that the break-even efficiency has also increased at a more or less equivalent rate. This could have the effect of eliminating all but the most efficient producers all at once.

IV. Conclusion

Technological progress has already brought down the cost of mining by orders of magnitude. The irony is that as competition to produce bitcoins induces more and more technological progress to increase efficiency and create competitive advantages, it might also serve to reduce the market price of bitcoin. The difficulty adjustment mechanism serves to stabilize the predicted decrease in production cost, and the halving of the block reward will do the same.

Of course, subjective factors will also confer value and be expressed in the market price. Speculators and miners who hoard all or part of their production are one example. Others may seek to own bitcoin for its decentralized nature and other unique features of anonymity, low transaction costs and security. Moreover, new innovations in using the blockchain technology, which bitcoin relies on, for non-bitcoin applications such as validation, verification and proof of ownership are sure to have value. These so-called Bitcoin 2.0 applications rely on the validation work of miners to secure non-bitcoin transactions and may prove to be more valuable than bitcoin in and of itself.

References

[1] Garcia, David, et al. *The digital traces of bubbles: feedback cycles between socio-economic signals in the Bitcoin economy.* Journal of The Royal Society Interface 11.99 (2014): 20140623.

[2] Hanley, Brian P. *The false premises and promises of Bitcoin.* arXiv preprint arXiv:1312.2048 (2013).

[3] Hayes, Adam, *What Factors Give Cryptocurrencies Their Value: An Empirical Analysis* (2014). Available at SSRN: http://ssrn.com/abstract=2579445

[4] Hayes, Adam, *The Decision to Produce Altcoins: Miners' Arbitrage in Cryptocurrency Markets* (March 16, 2015). Available at SSRN: http://ssrn.com/abstract=2579448

[5] Woo, David, et al. *Bitcoin: a first assessment.* FX and Rates (2013).

[6] Yermack, David. *Is Bitcoin a real currency? An economic appraisal.* No. w19747. National Bureau of Economic Research, 2013.

14 Imputed using market observations and equation (7).

15 Lags in difficulty adjustment over time may result in the actual halving date occurring somewhat prior to or after 4 years.

16 The change in block reward will have no impact on difficulty. Rather, less BTC/day will be found given the same difficulty.